Orbital Mechanics

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Problem 1 (**Satellite in Circular Orbit**)**.** Consider a satellite of mass m moving along a circular orbit with radius R around the Earth.

1. Determine the orbital velocity v_R and the period T of the satellite.
2. Calculate the total mechanical energy and the angular momentum of the satellite.
3. Determine the value of R for a geostationary orbit.

Now assume that the orbit is elliptical, with semi-major axis a and eccentricity ϵ.

4. Find v_m and v_M, the minimum and maximum speeds during the motion.
5. Determine the orbital period T.
6. Show that the following relation holds:
$$a = \frac{T}{2\pi}\sqrt{v_m v_M}.$$
7. Compute the total mechanical energy.

Problem 2 (**Orbital Maneuver**)**.** Consider the satellite described in the previous problem. Suppose it receives an instantaneous impulse of magnitude $\Delta v = \beta v_R$ in a specific direction. Determine the new value of the eccentricity and the minimum value of β required for the satellite to escape the gravitational field of the Earth in the following cases:

1. impulse in the tangential direction,
2. impulse in the radial direction,
3. impulse perpendicular to the orbital plane.

Problem 3 (**Earth Pendulum**)**.** Consider again a satellite in a circular orbit around the Earth.

1. What is the minimum possible orbital radius? How much time is required to complete one revolution?

Now assume that a tunnel is dug through the Earth, following the longest possible chord.

2. If an object is dropped into this tunnel, how much time will it take to return to the starting point?

3. How can this analogy between two different types of motion be explained?

Problem 4 (**Missile Toward the Equator**)**.** A missile is launched from the North Pole with the first cosmic velocity, and follows a trajectory that causes it to fall at a point located on the equator.

1. Determine the semi-major axis of the orbit followed by the missile.
2. What is the maximum height above the surface of the Earth reached by the missile?
3. Calculate the total flight time.

Problem 5 (**Time Until Collision**)**.** Consider two masses m_1 and m_2 initially at rest and separated by a distance d. Beginning at time $t = 0$, the masses move toward one another due to their mutual gravitational attraction. Determine the time required for the two masses to collide.

Problem 6 (**Shrinking Planet** — used in the Physics Team Competition)**.** A satellite is initially in a circular orbit with radius R_0 around a planet of mass M. Suppose the mass of the planet is suddenly reduced by a fraction f. Determine the maximum distance from the planet's center reached by the satellite in the new orbit.

Problem 7 (**Focus Transfer** — used in the Physics Team Competition)**.** A particle moves along an elliptical orbit with eccentricity $\epsilon < 1$, under the influence of gravitational attraction from a body located at one focus of the ellipse. When the particle reaches the point of maximum velocity, the attracting body is instantaneously transferred to the opposite focus. Determine the eccentricity of the new orbit.